EE565:Mobile Robotics
Lecture 6

Welcome

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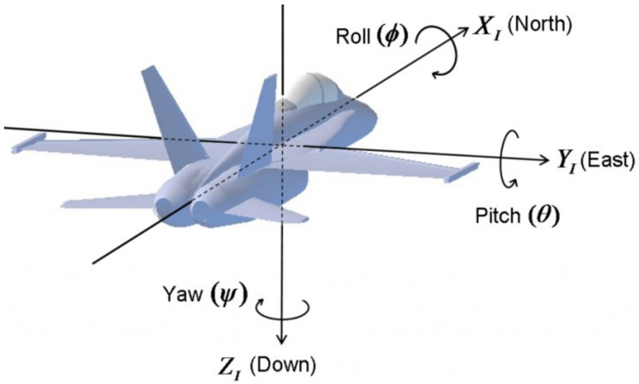
Announcement

- Mid-Term Examination # 1 (25%)
  - Understand basic wheel robot kinematics, common mobile robot sensors and actuators knowledge.
  - Understand and able to apply various robot motion and sensor models used for recursive state estimation techniques.
  - Demonstrate Inertial/visual odometeric techniques for mobile robots pose calculations.
- Date: **Wednesday** 4th March 2015
- Time: **1700-1800**
- Venue: **SDSB 203!!!**
- Format: Objective type
- Curriculum: Class and Lab Lecture Slides

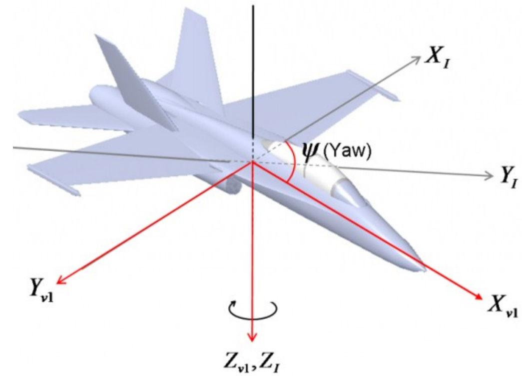
Today's Objectives

- Inertial sensors models
  - Euler Angle Representation
  - Accelerometer
    - Inertial Odometry
  - Gyroscope
    - Euler Angle Rate
  - Magnetometer
    - Earth Magnetic Field
  - GPS
    - Geodetic to Cartesian Coordinates

Euler Angles

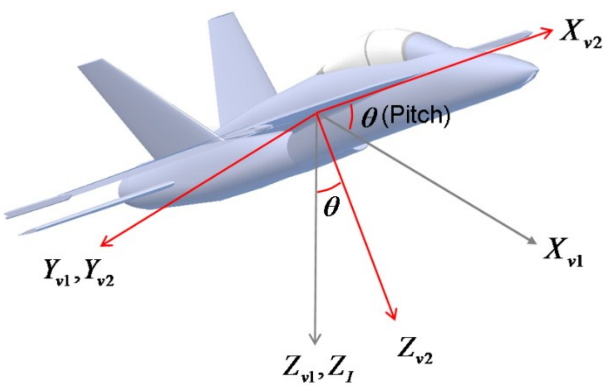


Inertial Frame



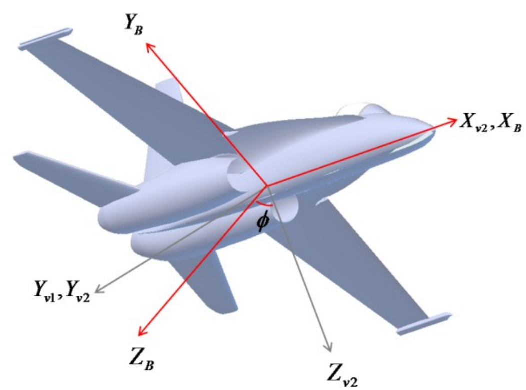
Vehicle-1 Frame

$$R_I^{v1}(\psi) = \begin{pmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



Vehicle-2 Frame

$$R_{v1}^{v2}(\theta) = \begin{pmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{pmatrix}$$



Body Frame

$$R_{v2}^{B}(\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{pmatrix}$$

Euler Angles (Cont.)

- When using Euler angles then a defined sequence, Yaw(ψ) →*Pitch*(θ) →*Roll*(ϕ), is used to transform body from one orientation to another orientation in 3D space. For example the transformation from inertial frame to body frame using Euler notation is

Inertial frame → $R_I^{v1}(\psi)$ → Vehicle-1 frame → $R_{v1}^{v2}(\theta)$ → Vehicle-2frame → $R_{v2}^{B}(\phi)$ → Body frame

$$R_I^B(\phi,\theta,\psi) = R_{v2}^B(\phi)\cdot R_{v1}^{v2}(\theta)\cdot R_I^{v1}(\psi)$$

- Similarly the transformation from body frame to inertial frame is

$$R_B^I(\phi,\theta,\psi) = R_I^{v2}(-\psi)\cdot R_{v1}^{v2}(-\theta)\cdot R_{v2}^B(-\phi)$$

Rotation Matrices

$$R_X(\phi) = R_{v2}^{B}(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mathrm{c}(\phi) & \mathrm{s}(\phi) \\ 0 & -\mathrm{s}(\phi) & \mathrm{c}(\phi) \end{bmatrix}$$

$$R_Y(\theta) = R_{v1}^{v2}(\theta) = \begin{bmatrix} \mathrm{c}(\theta) & 0 & -\mathrm{s}(\theta) \\ 0 & 1 & 0 \\ \mathrm{s}(\theta) & 0 & \mathrm{c}(\theta) \end{bmatrix}$$

$$R_z(\psi) = R_{I}^{v1}(\psi) = \begin{bmatrix} \mathrm{c}(\psi) & \mathrm{s}(\psi) & 0 \\ -\mathrm{s}(\psi) & \mathrm{c}(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Where's $\boldsymbol{cos(\theta) = c(\theta)}$ and $\boldsymbol{sin(\theta) = s(\theta)}$

Euler Angles (Cont.)

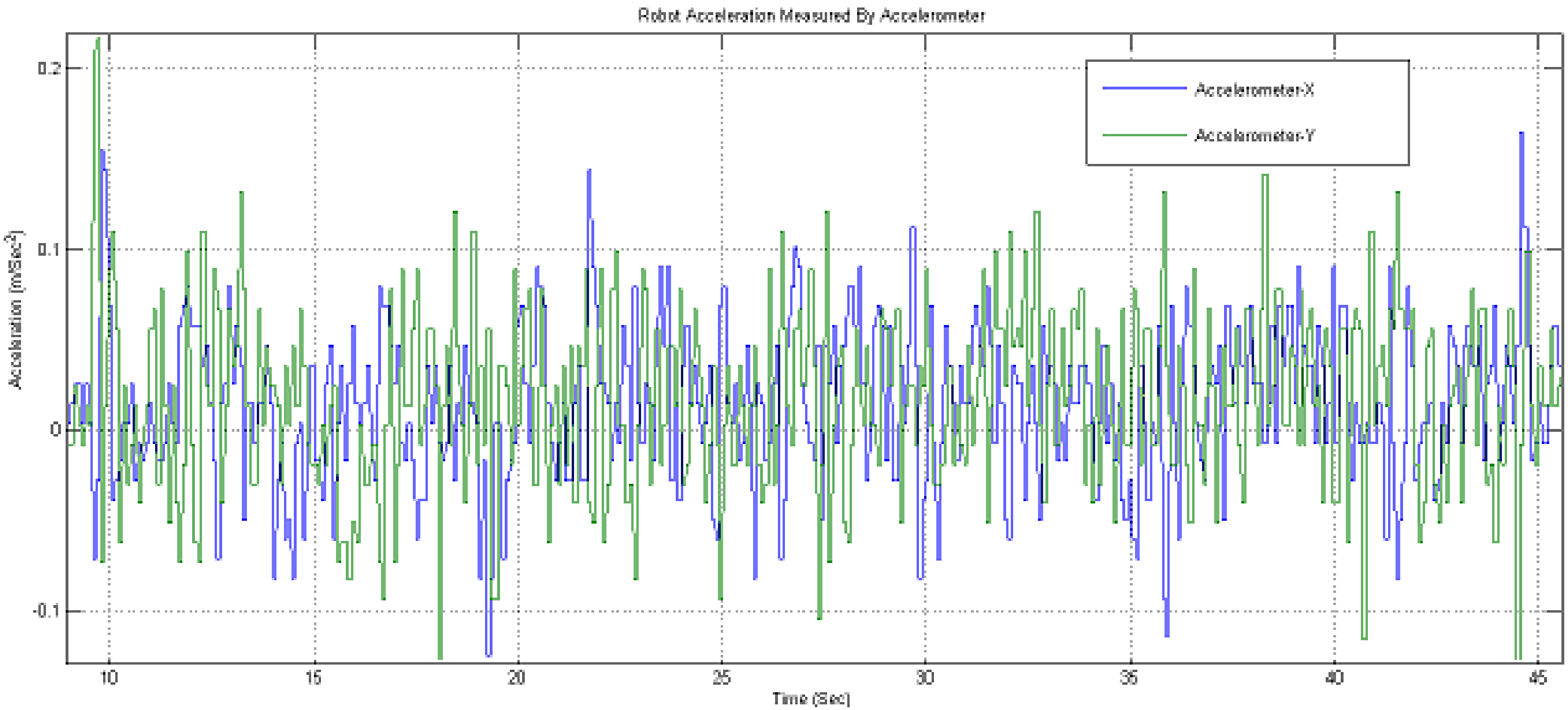
$$R_I^B(\phi,\theta,\psi) = \begin{bmatrix} \mathrm{c}(\psi)\,\mathrm{c}(\theta) & \mathrm{c}(\theta)\,\mathrm{s}(\psi) & -\,\mathrm{s}(\theta) \\ \mathrm{c}(\psi)\,\mathrm{s}(\phi)\,\mathrm{s}(\theta) - \mathrm{c}(\phi)\,\mathrm{s}(\psi) & \mathrm{c}(\phi)\,\mathrm{c}(\psi) + \mathrm{s}(\phi)\,\mathrm{s}(\psi)\,s(\theta) & \mathrm{c}(\theta)\,\mathrm{s}(\phi) \\ s(\phi)s(\psi) + c(\phi)c(\psi)s(\theta) & c(\phi)s(\psi)s(\theta) - c(\psi)s(\phi) & c(\phi)c(\theta) \end{bmatrix}$$

$$R_B^I(\phi,\theta,\psi) = \begin{bmatrix} \mathrm{c}(\psi)\,\mathrm{c}(\theta) & \mathrm{c}(\psi)\,\mathrm{s}(\phi)\,\mathrm{s}(\theta) - \mathrm{c}(\phi)\,\mathrm{s}(\psi) & s(\phi)s(\psi) + c(\phi)c(\psi)s(\theta) \\ \mathrm{c}(\theta)\,\mathrm{s}(\psi) & \mathrm{c}(\phi)\,\mathrm{c}(\psi) + \mathrm{s}(\phi)\,\mathrm{s}(\psi)\,s(\theta) & c(\phi)s(\psi)s(\theta) - c(\psi)s(\phi) \\ -\,\mathrm{s}(\theta) & \mathrm{c}(\theta)\,\mathrm{s}(\phi) & c(\phi)c(\theta) \end{bmatrix}$$

$$R_I^B(\phi,\theta,\psi) = \left(R_B^I(\phi,\theta,\psi)\right)^T$$

Inertial Sensor Model

- We require sensor models to calibrate sensors in order to remove noise from sensor measurements thus enhances the accuracy of sensor measurements

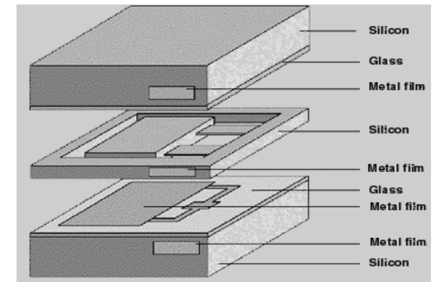


Measurement Errors

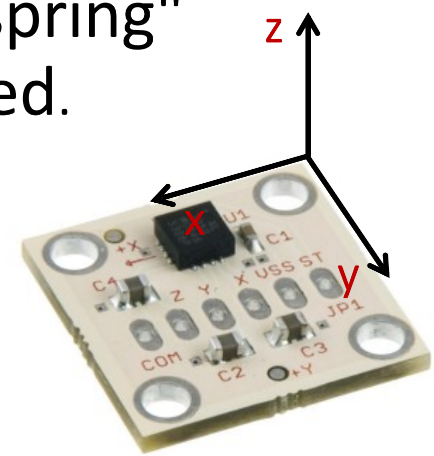
- Sensor errors are the deviations of sensor measurements from its expected output.
- Systematic errors: periodic, able to be removed by calibration process
- Stochastic errors: random

Accelerometer

- Measures all external forces acting upon them (including gravity)
- To obtain inertial acceleration (due to motion alone), gravity must be subtracted

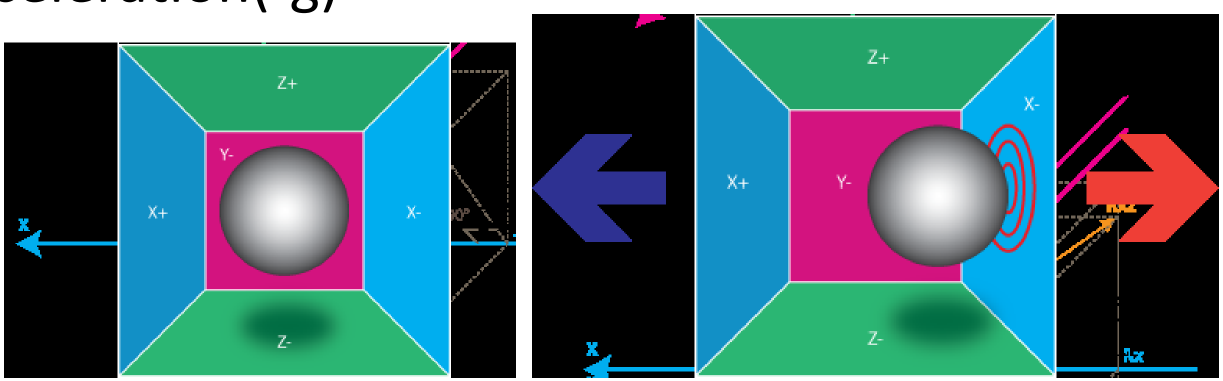


- Accelerometers behave as a damped mass on a spring. Acceleration causes displacement of this "spring" proportional to the acceleration experienced.
  - This room = your weight = 1g
  - Bugatti Veyron, 0 to 100Km/h in 2.4s= 1.55g
  - Space Shuttle reentry & launch = 3g
  - Max experienced by a human* = 46.2g
  - Death or extensive & severe injuries= +50g



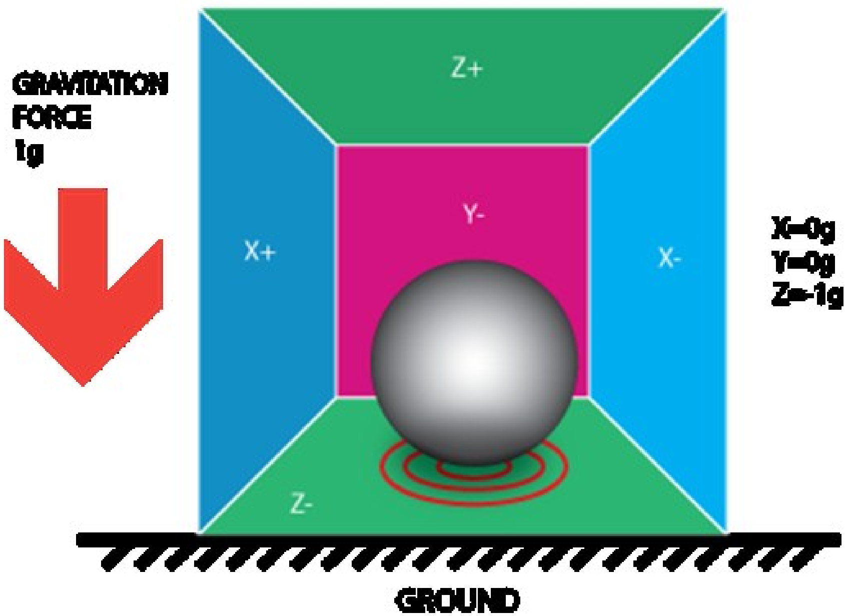
Accelerometer (Cont.)

- It measures acceleration indirectly by the use of force applied on its body.
- It measures force in opposite direction of acceleration.
- Imagine a cube with pressure sensitive walls and a sphere inside it
  - In space the ball floats in the middle of box
  - If we accelerate(g) to left, the right wall measure acceleration(-g)



Accelerometer (Cont.)

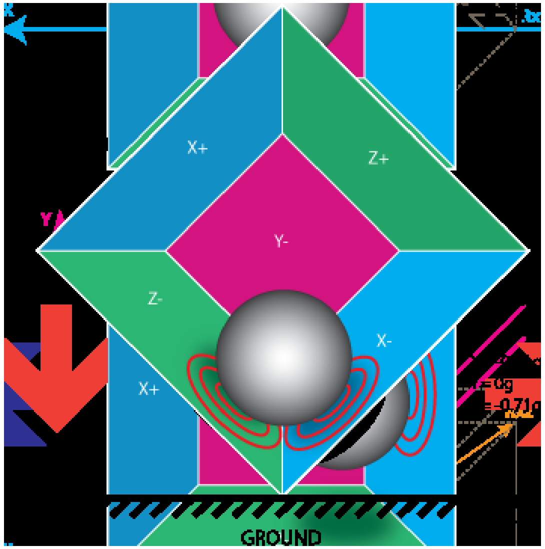
- If we are on earth the ball will fall on the **Z-** wall and will apply a force of 1g (gravitation force) on the bottom wall

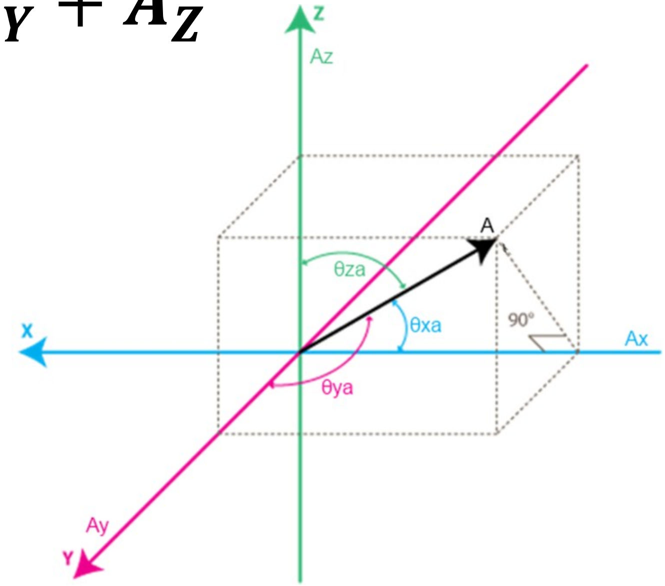


Accelerometer (Cont.)

- If we rotate the accelerometer an angle of 45 the X and Z axis will measure the acceleration.
- If the accelerometer is stationary then the measured acceleration due to gravity can be calculated as follows

$$A^2 = A_X^2 + A_Y^2 + A_Z^2$$





Accelerometer Raw Measurements

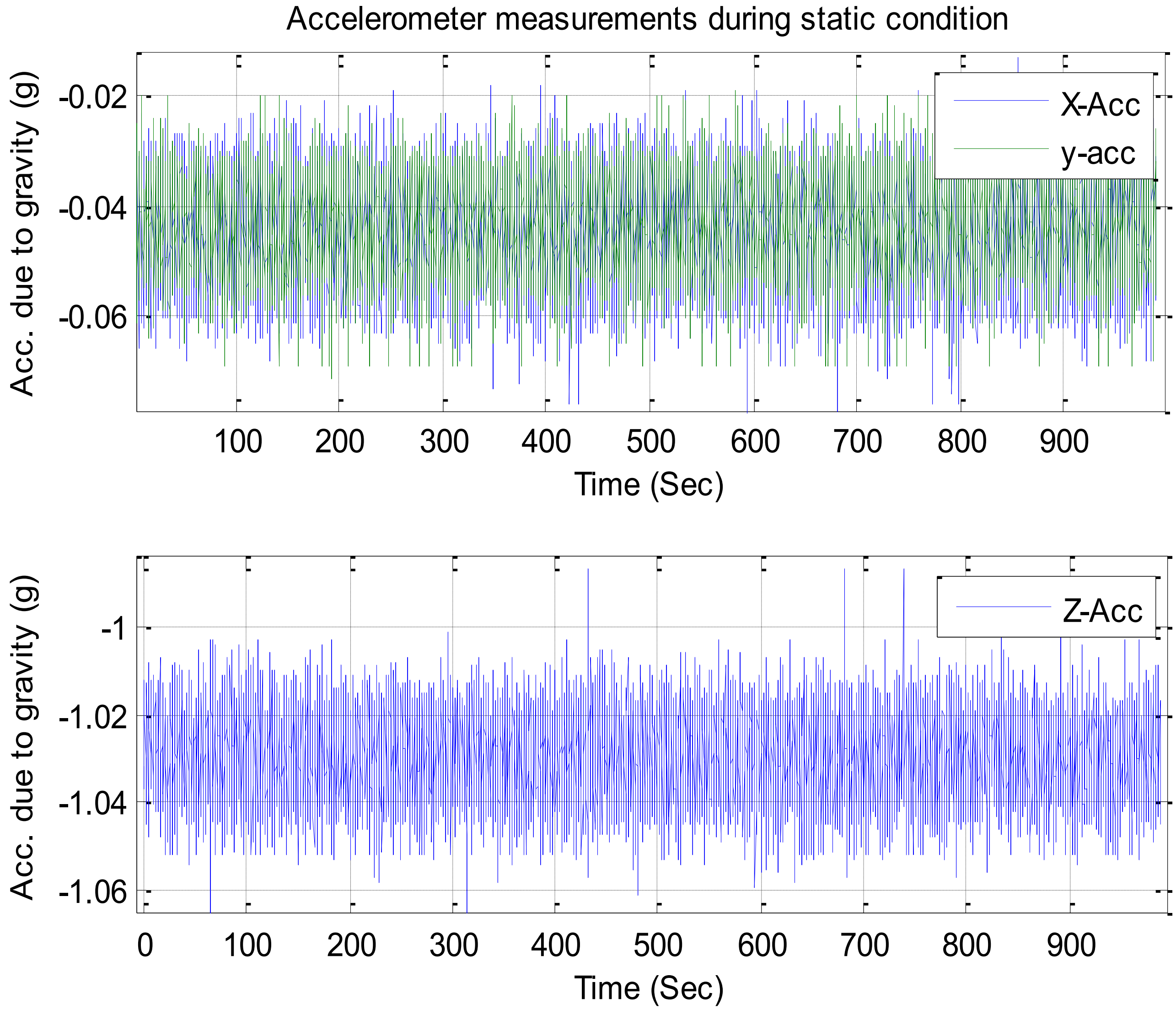
To convert the raw accelerometer's ADC value we use the following equation

$$\boldsymbol{A_X = \frac{\left(AdcValue_X \times \frac{V_{ref}}{2^{AdcBits} - 1}\right) - V_{g_0}}{Sensitivity} = \frac{(AdcValue - AdcValue_{g_0}) \times V_{ref}}{Sensitivity \times (2^{AdcBits} - 1)}}$$

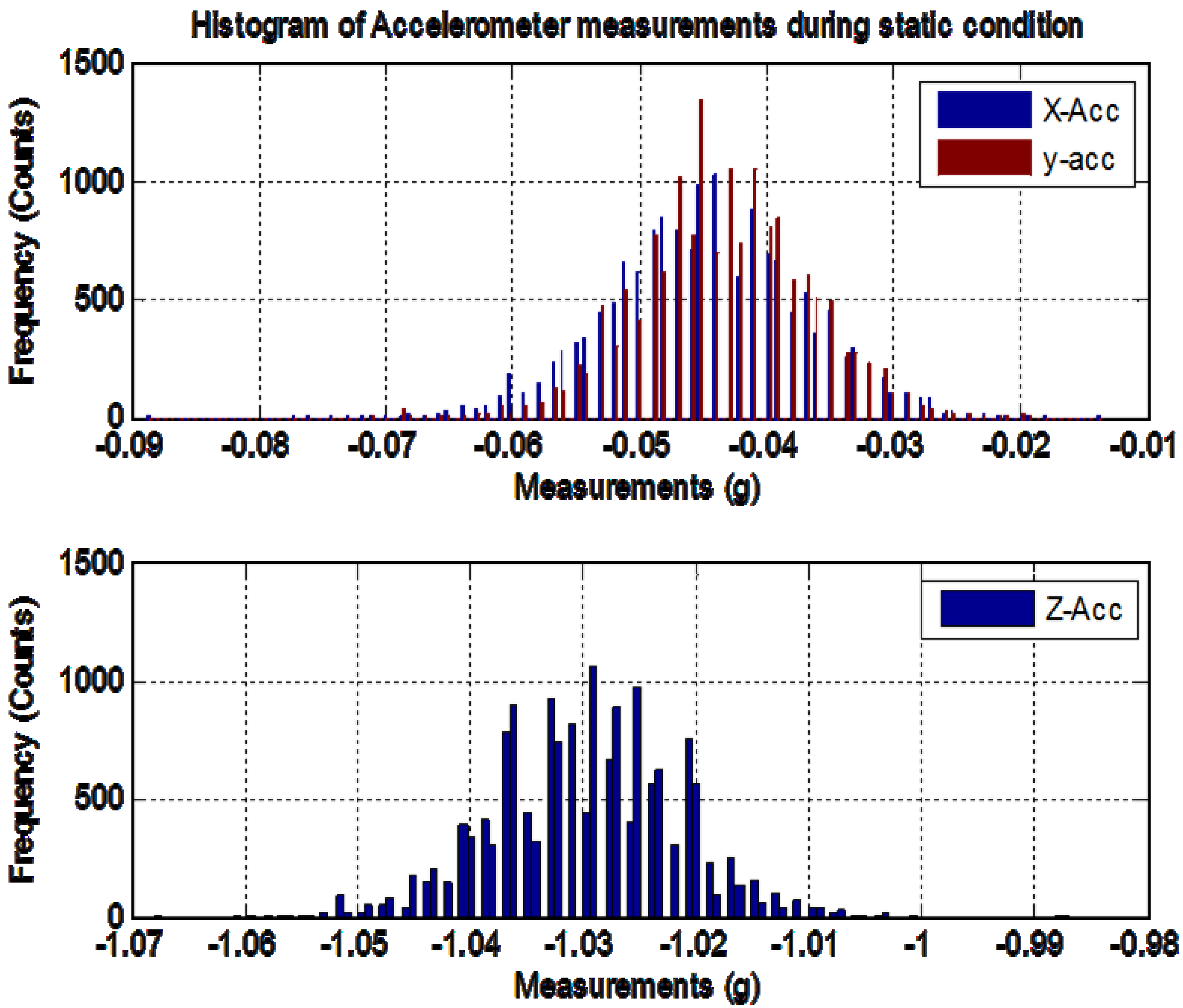
Assuming $AdcValue_x = 530$ $V_{ref} = 3.3V$, 10bits ADC, $g_0 = 1.65V$, ADC $Sensitivity = \frac{0.4785V}{g}$

$$\frac{(530 - 512) \times 3.3}{0.4785 \times 1023} = 0.1213g$$

Accelerometer Measurement Noise



Accelerometer Noise Statistics



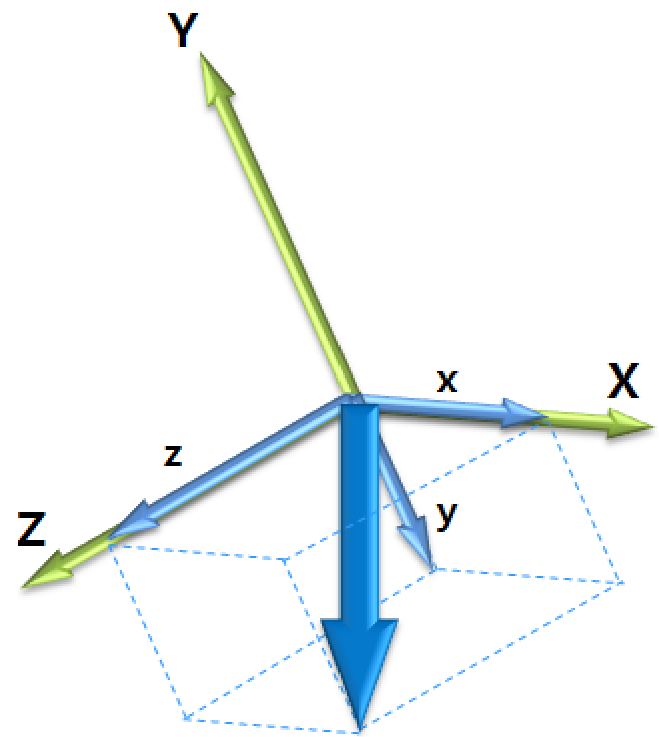
Euler Angles by Accelerometer

- Gravity vector is perpendicular to the horizontal plane made by the earth surface. Therefore, **the gravity vector can be used to determine the roll and pitch angle of the horizontal plane**.
- Since the horizontal plan can be rotated to any yaw angle with respect to gravity vector direction, therefore, the yaw angle cannot be uniquely determined.
- The accelerometer readings are in body frame and the gravity vector is in inertial frame.
- If we assume that no external acceleration is acted on the accelerometer then the gravity sensed by the sensor in accelerometer body frame is

$$A_b = \left(a_x\, a_y\, a_z\right)^T$$

where the gravity in the inertial/reference frame is

$$A_i = (0\ 0\ g)^T.$$



Euler Angles by Accelerometer (Cont.)

- The relationship between A_b and A_i is as follows

$$A_b = C_i^b(\theta, \phi, \psi) \cdot A_i$$
$$A_b = R_x(\phi) \cdot R_y(\theta) \cdot R_z(\psi) \cdot A_i$$

- Accelerometer measures the components of acceleration due to gravity of an oriented sensor.
- Since we know the gravity vector, therefore the roll and pitch angles are measured by applying the inverse roll and pitch angle transformation to the accelerometer measurements

$$R_y(-\theta) \cdot R_x(-\phi) \cdot A_b = R_y(-\theta) \cdot R_x(-\phi) \cdot R_x(\phi) \cdot R_y(\theta) \cdot R_z(\psi) \cdot A_i$$

Euler Angles by Accelerometer (Cont.)

$$R_y(-\theta) \cdot R_x(-\phi) \cdot A_b = R_z(\psi) \cdot A_i$$
$$R_y(-\theta) \cdot R_x(-\phi) \cdot A_b = A_i$$

$$\begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix} \cdot \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}$$

$$\begin{bmatrix} a_x \cos(\theta) + a_y \sin(\phi) \sin(\theta) + a_z \cos(\phi) \sin(\theta) \\ a_y \cos(\phi) - a_z \sin(\phi) \\ -a_x \sin(\theta) + a_y \sin(\phi) \cos(\theta) + a_z \cos(\phi) \cos(\theta) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}$$

Euler Angles by Accelerometer (Cont.)

$$\begin{bmatrix} a_x \cos(\theta) + a_y \sin(\phi) \sin(\theta) + a_z \cos(\phi) \sin(\theta) \\ a_y \cos(\phi) - a_z \sin(\phi) \\ -a_x \sin(\theta) + a_y \sin(\phi) \cos(\theta) + a_z \cos(\phi) \cos(\theta) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}$$

Solving the above equationfor $\phi\ and\ \theta$ results into

$$\phi = -\arctan\left(\frac{a_y}{a_z}\right)$$

$$\theta = -\arctan\left(\frac{a_x}{\sqrt{a_y^2 + a_z^2}}\right) = -\arctan\left(\frac{a_x}{a_y \sin(\phi) + a_z \cos(\phi)}\right)$$

Inertial Odometry by Accelerometer

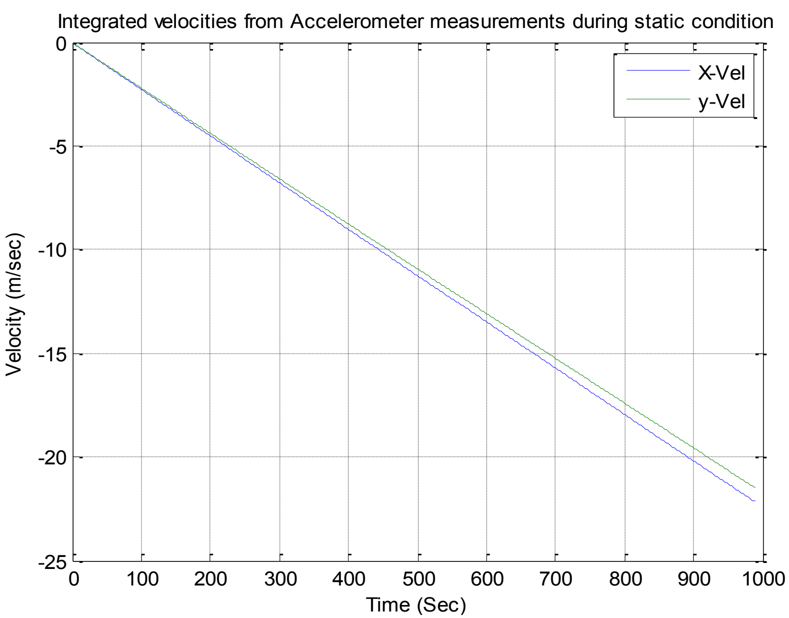
- To perform inertial odometry the linear acceleration which is being applied on the body has to be determined.
- Since the accelerometer measure the acceleration due to gravity in addition to acceleration being applied on the body, a normal force is being applied on the accelerometer which keep it moving towards the center of earth.
- Therefore, In order to extract the linear acceleration from the accelerometer measurements, first the measurements are rotated by the accelerometer yaw-pitch-roll angles so that they are in the inertial frame and then gravity vector is being added to it which prevent the accelerometer from moving toward center of earth

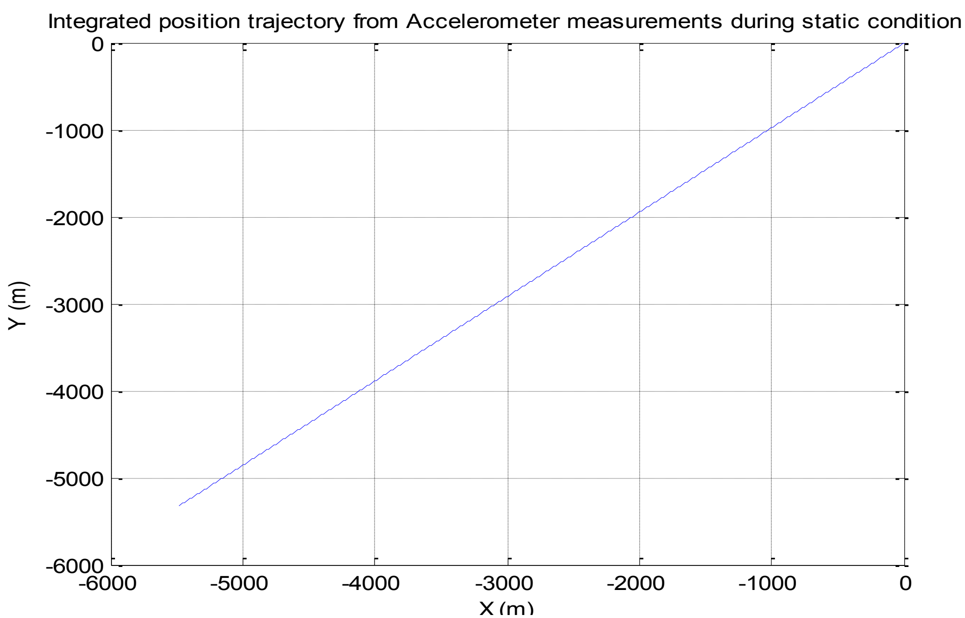
Inertial Odometry

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}_i = \begin{bmatrix} c(\psi)\,c(\theta) & c(\psi)\,s(\phi)\,s(\theta) - c(\phi)\,s(\psi) & s(\phi)s(\psi) + c(\phi)c(\psi)s(\theta) \\ c(\theta)\,s(\psi) & c(\phi)\,c(\psi) + s(\phi)\,s(\psi)\,s(\theta) & c(\phi)s(\psi)s(\theta) - c(\psi)s(\phi) \\ -s(\theta) & c(\theta)\,s(\phi) & c(\phi)c(\theta) \end{bmatrix} \cdot \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}_b + \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}$$

- Position can be estimated using **the rectangular integration** method as follows

$$v_k = v_{k-1} + a_k \cdot \Delta t$$
$$p_k = p_{k-1} + v_k \cdot \Delta t$$





Inertial Odometry

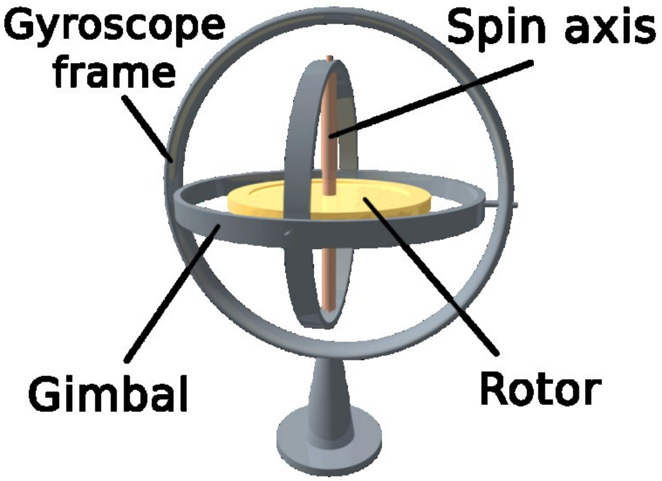
- In addition to the accelerometer noise errors the biggest error terms come from the **orientation** (transformation matrix) **errors**.
- If the accelerometer measurements are assumed to be free from noise then the errors in position and velocity due to the errors in the orientation errors can be surmised from the following table

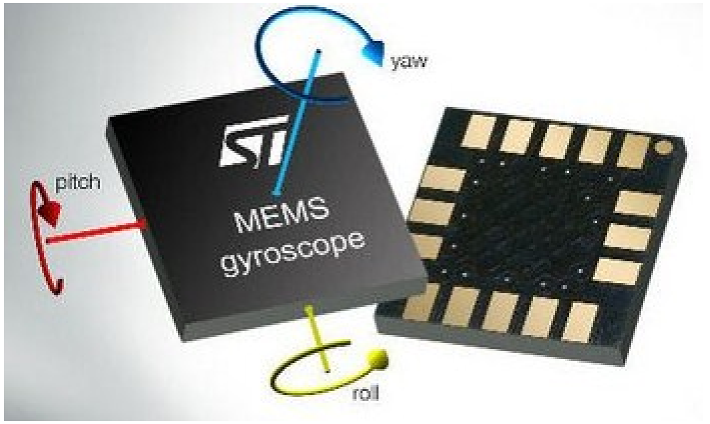
Inertial Odometry

Angle Error (degrees)	Acceleration Error (m/s/s)	Velocity Error (m/s) @ 10 seconds	Position Error (m) @ 10 seconds	Position Error (m) @ 1 minute	Position Error (m) @ 10 minutes	Position Error (m) @ 1 hour
0.1	1.71	0.2	1.7	62	6157	221.6 E 3
0.5	8.55	0.9	8.6	308	30.8 E 3	1.1 E 6
1.0	17.11	1.7	17.1	615	61.6 E 3	2.2 E 6
1.5	25.66	2.6	25.6	924	92.4 E 3	3.3 E 6
2.0	34.22	3.4	34.2	1232	123.2 E 3	4.4 E 6

Gyroscope

- A gyroscope is a device for measuring or maintaining orientation, based on the principles of conservation of angular momentum
- Measures orientation (standard gyro) or angular velocity (rate gyro, needs integration for angle)





Gyroscope Raw Measurements

- Gyroscope measures the rate of changes of the angles. For instance a 3-axes gyroscope will measure the rotation rate about the X, Y and Z axes.
- In practice we'll get an ADC value that need to converted to $\frac{\theta^\circ}{\text{Sec}}$ using the following formula

$$\dot{\boldsymbol{\theta}} = \frac{(\boldsymbol{AdcValue} - \boldsymbol{AdcValue_0}) \times \boldsymbol{V_{ref}}}{\boldsymbol{Sensitivity} \times (\boldsymbol{2^{AdcBits}} - \boldsymbol{1})}$$

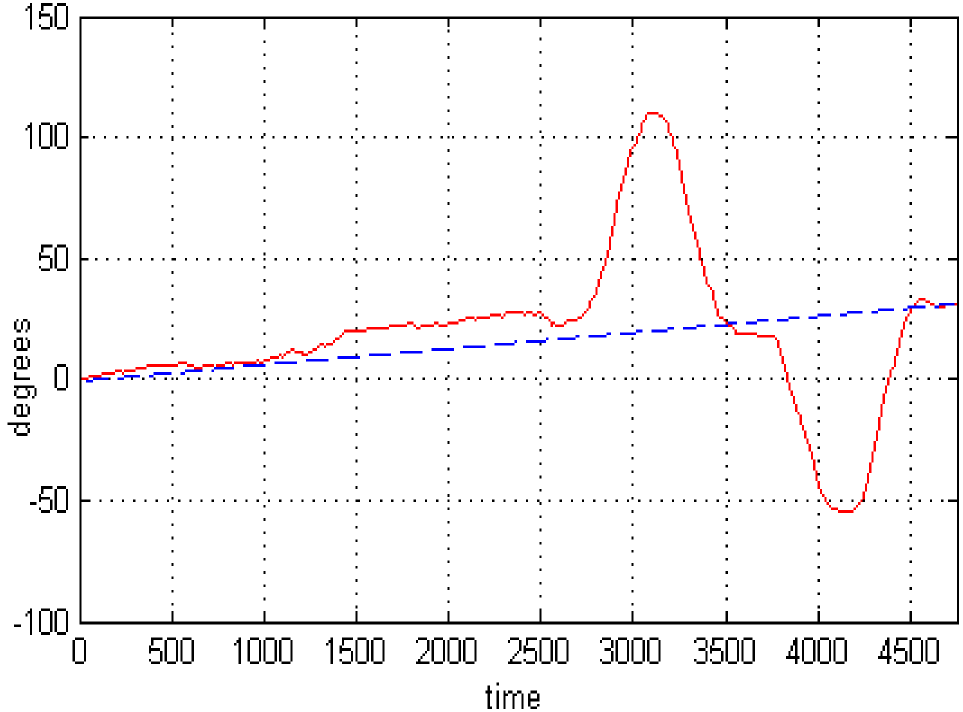
Gyroscope Raw Measurements

- $AdcValue_0$ is the value which gyroscope outputs when it is not subjected to rotation
- Sensitivity: is the scale of measurements and expressed in $\mathrm{mV}/_{\circ/sec}$
- For example using a 10bits ADC with $V_{ref} = 3.3V, \mathrm{AdcValue}_0 = 512, \mathrm{Sensitivit} = 0.002\mathrm{V}/°/sec$ and $AdcValue = 571$

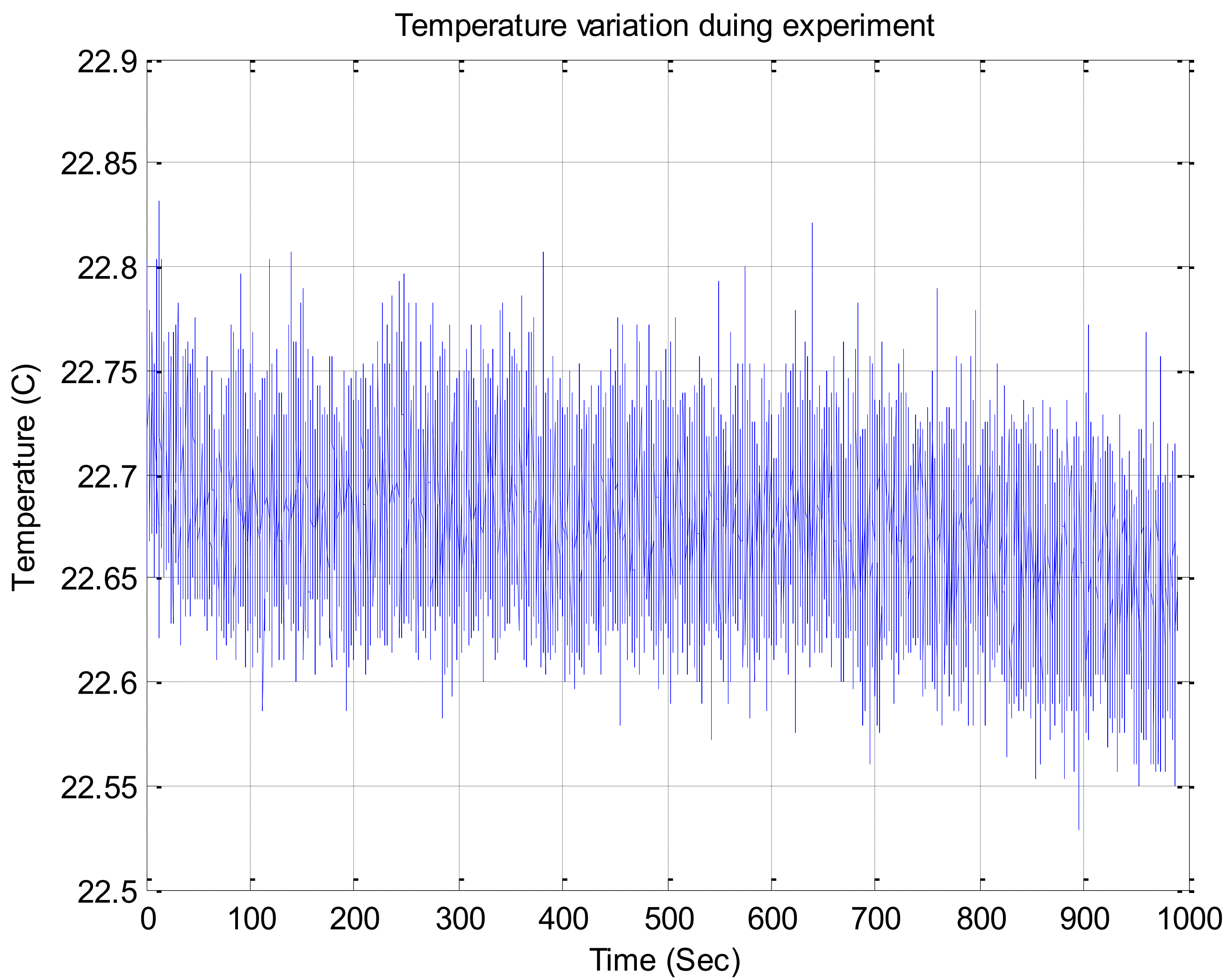
$$\dot{\boldsymbol{\theta}} = \frac{(\mathbf{571} - \mathbf{512}\,) \times \mathbf{3.3}}{\mathbf{0.002} \times \mathbf{1023}} = 95°/sec$$

Gyroscope Bias Drift

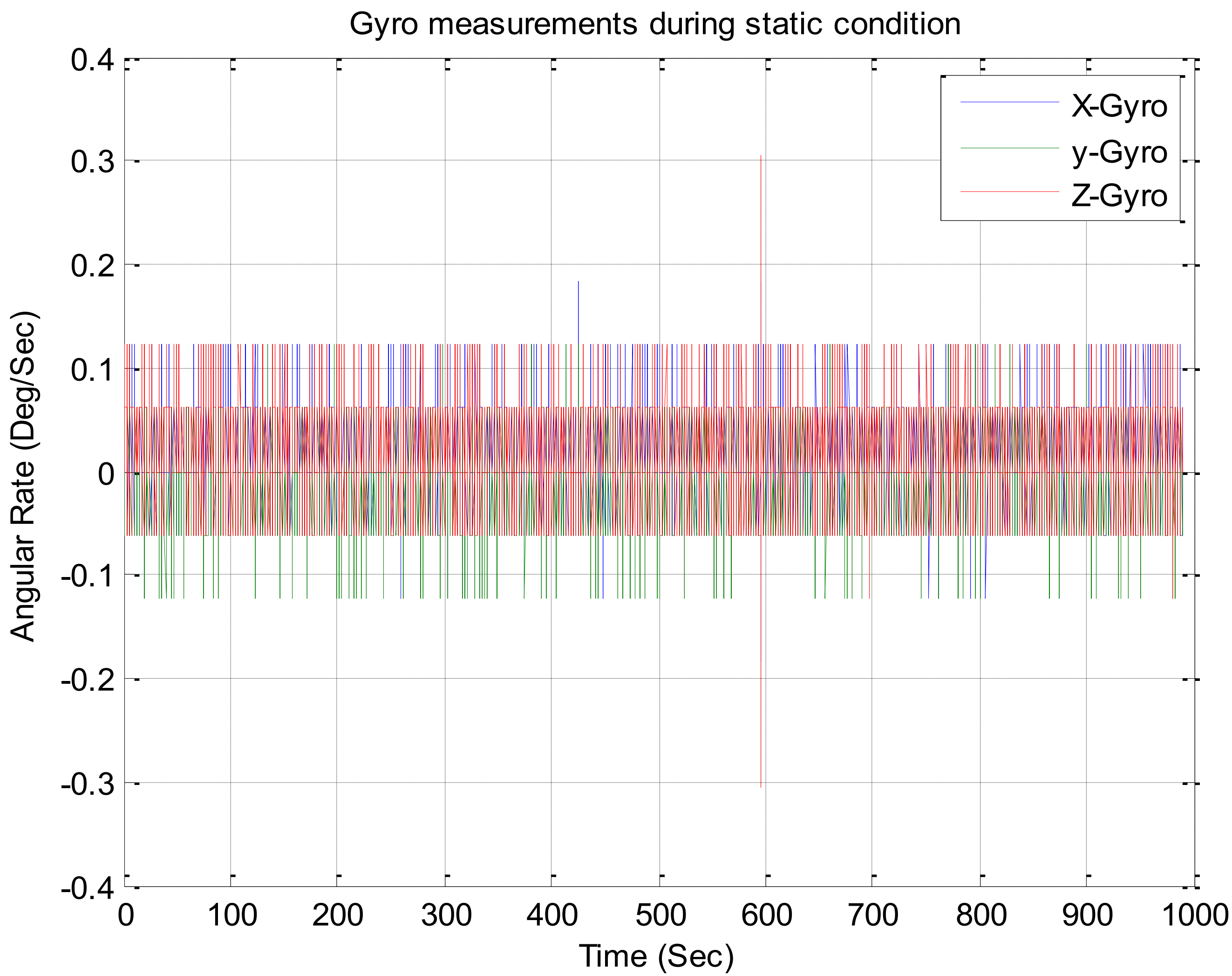
- Gyroscope measurements are integrated to get angle. The blue line represents the drift. The following data shows the bias **drift rate** of $\mathbf{24°/\boldsymbol{hr}}$



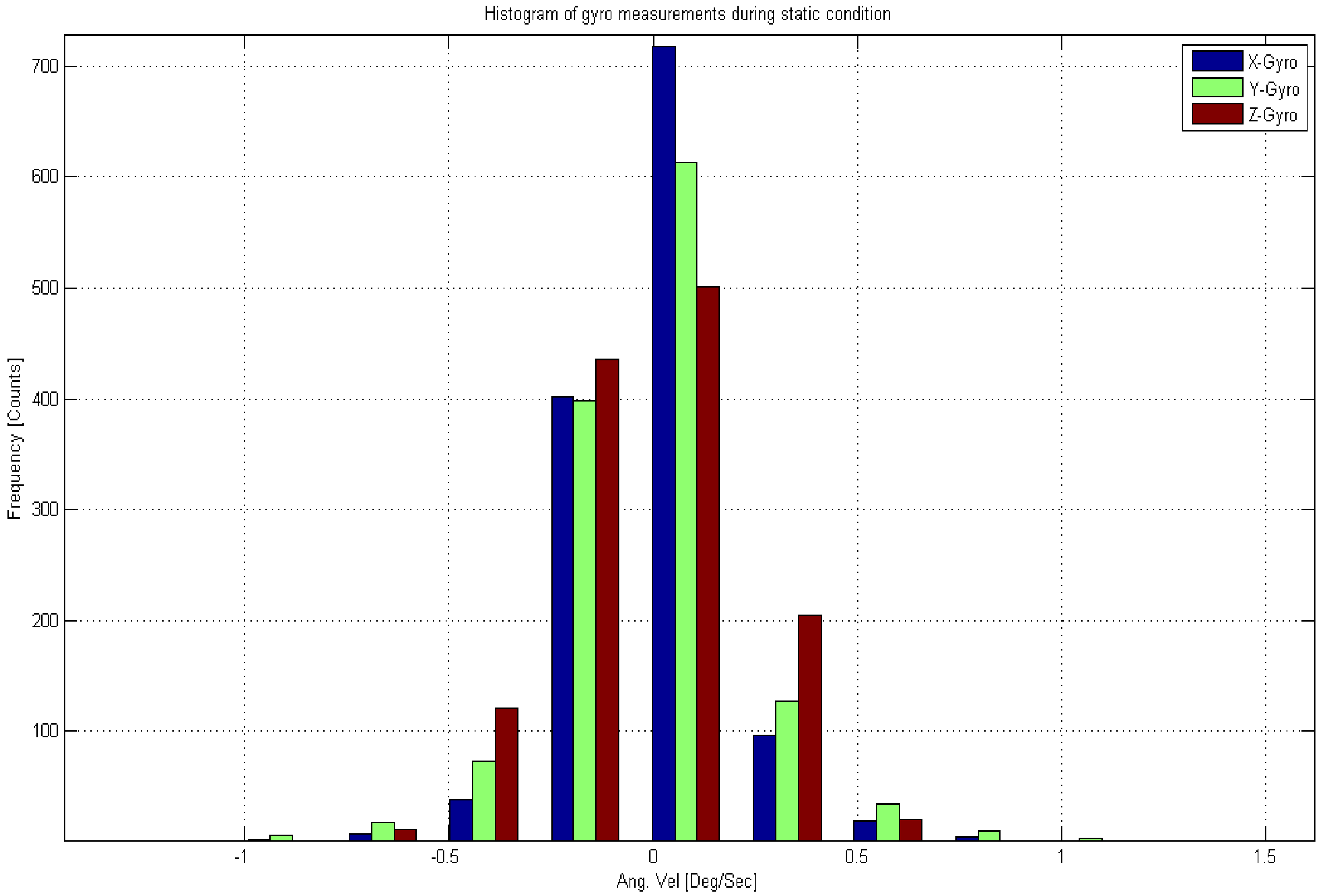
Gyroscope Temperature Drift



Gyroscope Measurements Noise

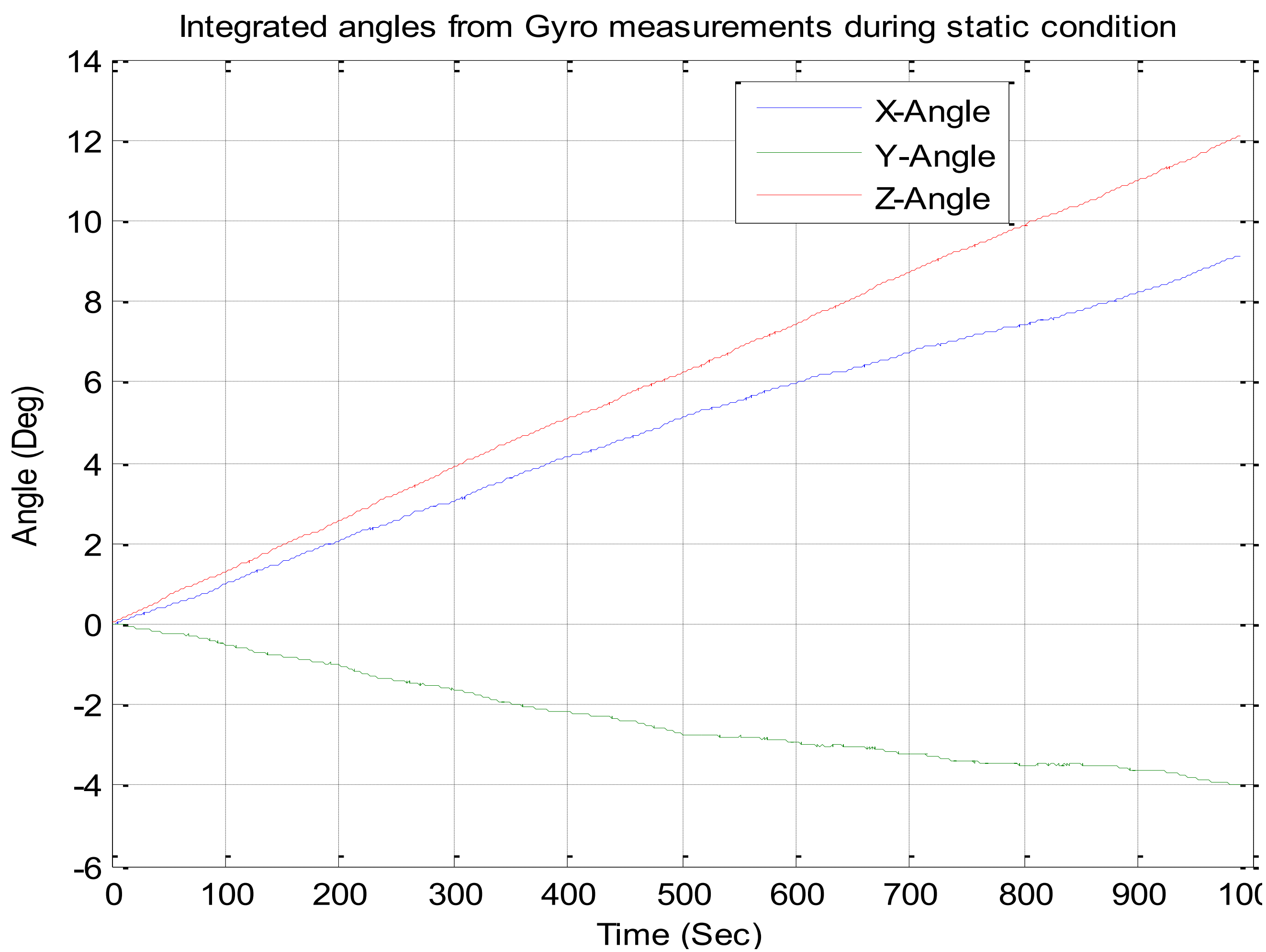


Gyroscope Noise Statistics



Gyroscope Integration

$$\theta_t = \theta_{t-1} + \dot{\theta}_t \cdot \Delta t$$



Euler Angular Rate Using Gyroscope

- The gyroscopes measurements **can't** be directly used because it measures body angular rates not the Euler angular rate.
- The gyroscope measurements can be converted into **Euler angular rate** using the following transformation

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi)\,/\cos(\theta) & \cos(\phi)\,/\cos(\theta) \end{bmatrix} \cdot \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

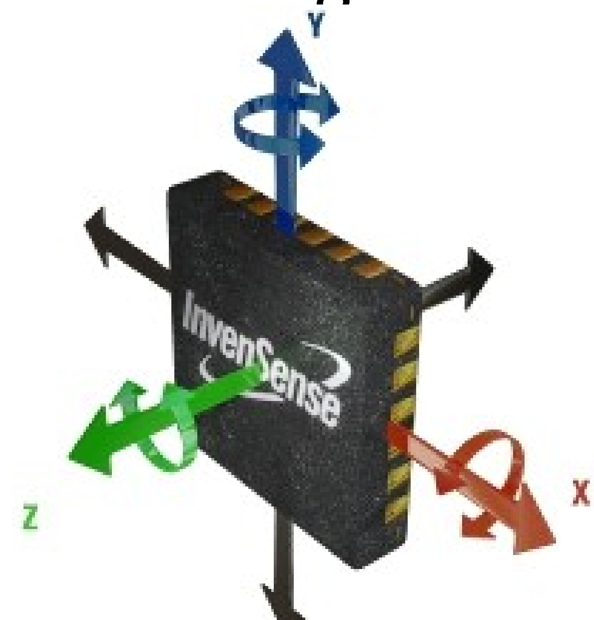
Euler Angular Rate Using Gyroscope

$$D(\phi, \theta, \psi) = \begin{bmatrix} 1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi)/\cos(\theta) & \cos(\phi)/\cos(\theta) \end{bmatrix}$$

- The above transformation matrix is obtained by
  - first rotating the z-axis gyro in inertial frame
  - then the y-axis gyro is rotated into vehicle-1 frame
  - And finally x-axis gyro is rotated in vehicle-2 frame

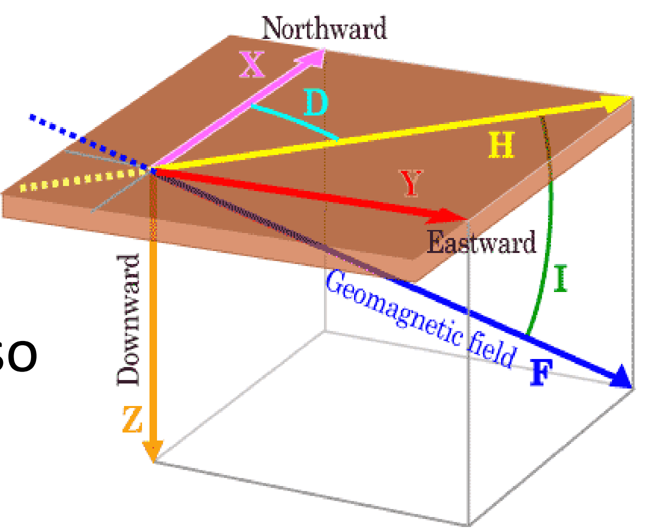
Magnetometer

- Compass invented by the Chinese in the 4th century, Carl Gauss invented the "magnetometer" in 1833. MEMS magnetometer measures the components of earth’s magnetic field intensity

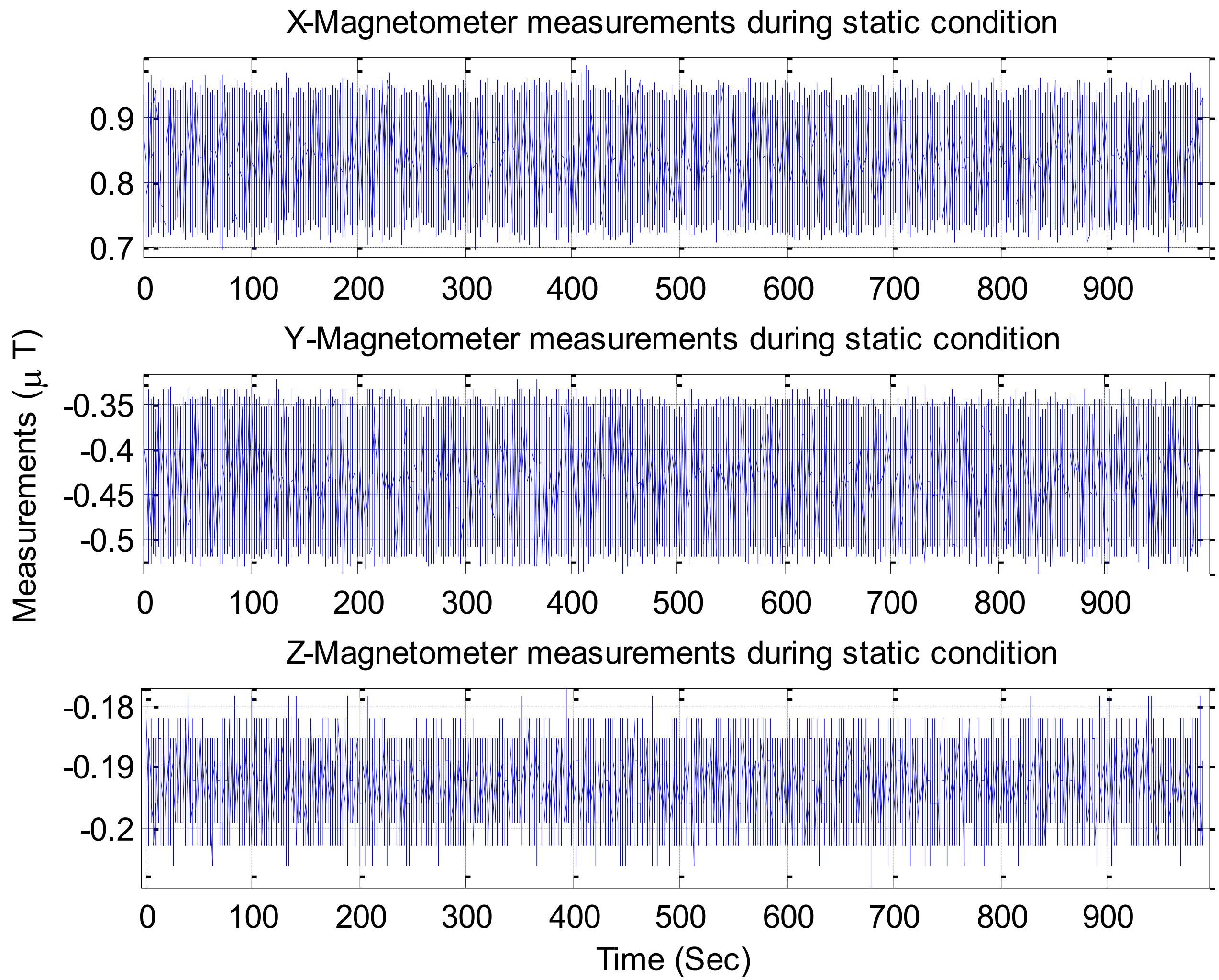


Earth Magnetic Field

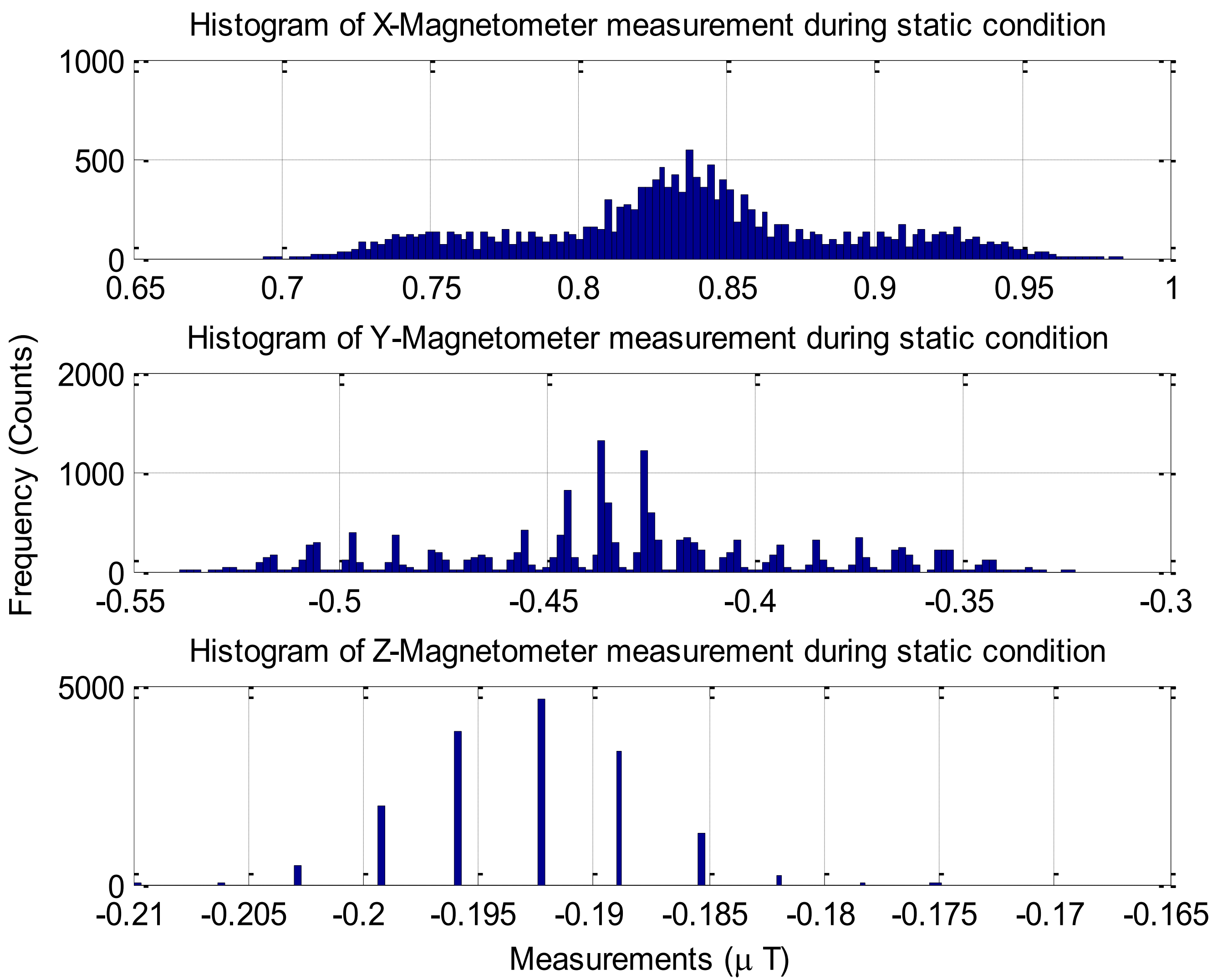
- The earth magnetic field intensity is 20μT - 70μT Gauss and has a component of magnetic field parallel to earth surface that always point North while the vertical component points toward the center of earth. The vertical component of magnetic field is not required to find magnetic north.
- The geomagnetic field has strength of F and made an inclination angle of I depending upon the latitude and longitude. The difference between geographic north and magnetic north has an declination angle D which is also location dependent



Magnetometer (Cont.)

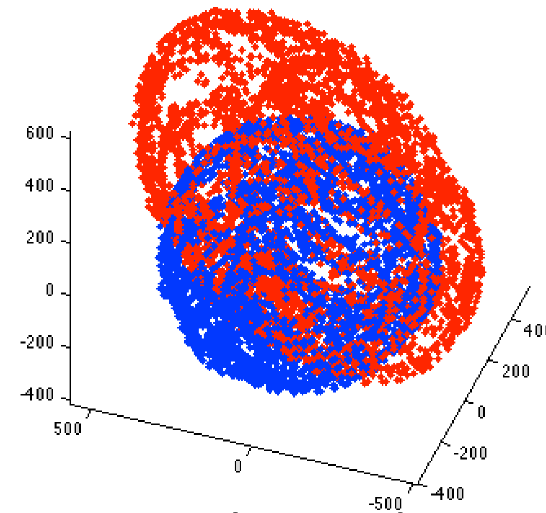


Magnetometer (Cont.)



Hard/Soft Iron Effects

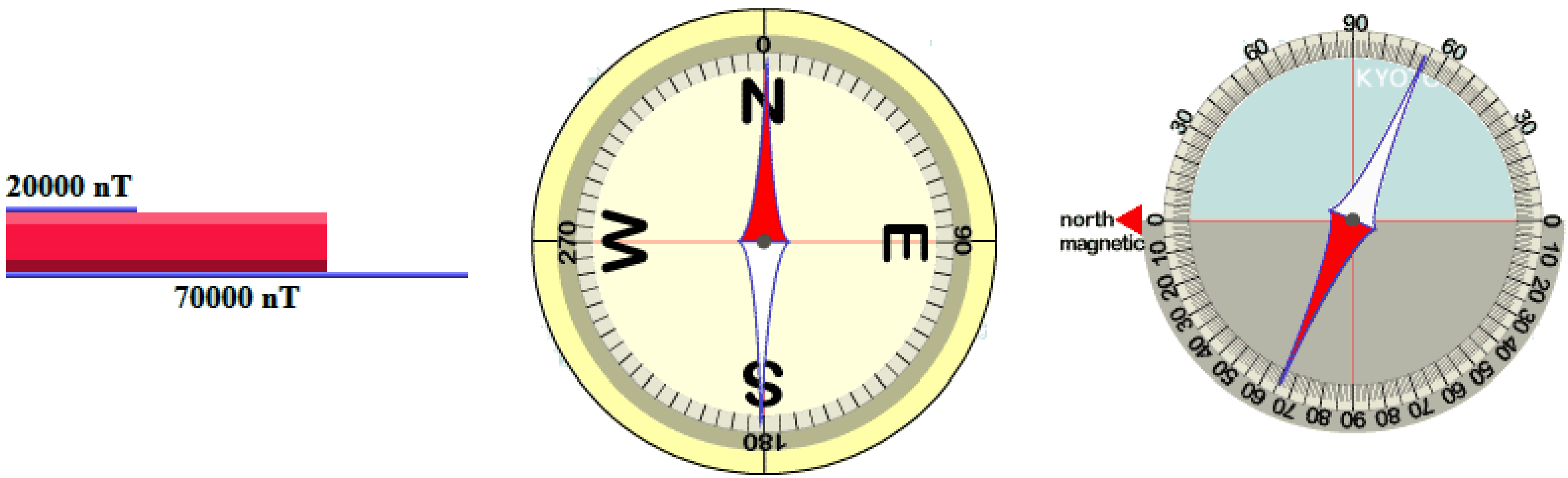
- When the x-axis of the magnetometer is pointing northward and downward then it experiences maximum geomagnetic field strength



- The accuracy of the magnetometer measurements depends on the stray magnetic field inside and around the magnetometer.
- The fixed part is called hard-iron effect and the part due to geomagnetic field is called soft-iron effect. The magnetic field strength and inclination angle depends upon the location (latitude and longitude) on the surface of earth.

Earth Magnetic Field

Geomagnetic elements Lat.: 50.883N Long.: 8.017E Altitude: 290.0m Year:2012 (Prediction) (IGRF-11)



Total Intensity (F): 48741.8 nT

Declination (D): 1.331°

Inclination (I): 66.282°

Northward (X): 19600.2 nT

Eastward (Y): 455.5 nT

Downward (Z): 44624.9 nT

Horizontal (H): 19605.5 nT

http://wdc.kugi.kyoto-u.ac.jp/igrf/point/index.html

Orientation Using Magnetometer

- It can be used to determine heading direction using the geomagnetic measurements

$$M_b = \left(m_x \; m_y \; m_z\right)^T$$

and the roll and pitch angle measured by the accelerometer.

Orientation Using Magnetometer

- The measurements of the magnetometer in sensor frame can be calculated as follows
$$M_b = R_x(\phi) \cdot R_y(\theta) \cdot R_z(\psi) \cdot M_i + V$$
- V is the **soft iron effects** due to the metallic objects near the magnetometer; the soft iron effects vector is independent from rotations because the objects rotate along magnetometer such as the PCB or casing.

Orientation Using Magnetometer

- The magnetometer readings are applied with inverse roll and pitch angles so that the measurements are only affected by the yaw angle i.e.

$$R_y(-\theta) \cdot R_x(-\phi) \cdot (M_b - V) = R_y(-\theta) \cdot R_x(-\phi) \cdot R_x(\phi) \cdot R_y(\theta) \cdot R_z(\psi) \cdot M_i$$

$$R_y(-\theta) \cdot R_x(-\phi) \cdot (M_b - V) = R_z(\psi) \cdot M_i$$

Orientation Using Magnetometer

$$\begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix} \cdot \begin{bmatrix} m_x - v_x \\ m_y - v_y \\ m_z - v_z \end{bmatrix} = \begin{bmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} B \cdot \cos(\delta) \\ 0 \\ B \cdot \sin(\delta) \end{bmatrix}$$

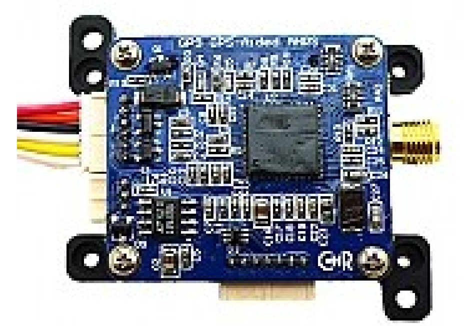
$$\begin{bmatrix} (m_x - v_x)\cos(\theta) + (m_y - v_y)\sin(\phi)\sin(\theta) + (m_z - v_z)\cos(\phi)\sin(\theta) \\ (m_y - v_y)\cos(\phi) - (m_z - v_z)\sin(\phi) \\ -(m_x - v_x)\sin(\theta) + (m_y - v_y)\sin(\phi)\cos(\theta) + (m_z - v_z)\cos(\phi)\cos(\theta) \end{bmatrix} = \begin{bmatrix} B \cdot \cos(\delta) \cdot \cos(\psi) \\ -B \cdot \cos(\delta) \cdot \sin(\psi) \\ B \cdot \sin(\delta) \end{bmatrix}$$

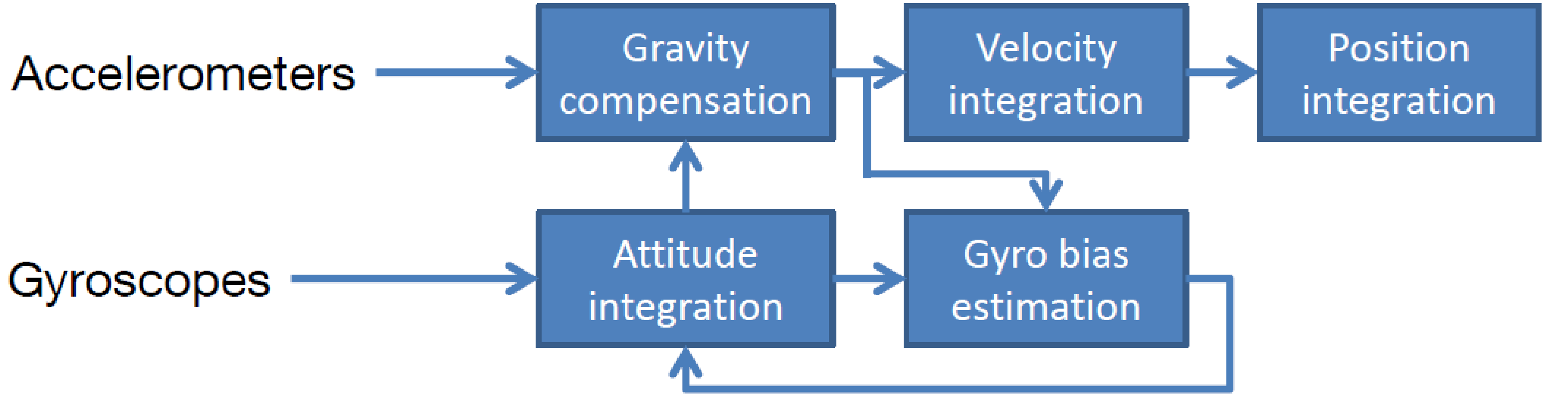
Dividing the y-component by the x-component gives us the yaw angle

$$\psi = \arctan\left(\frac{(m_z - v_z)\sin(\phi) - (m_y - v_y)\cos(\phi)}{(m_x - v_x)\cos(\theta) + (m_y - v_y)\sin(\phi)\sin(\theta) + (m_z - v_z)\cos(\phi)\sin(\theta)}\right)$$

IMU

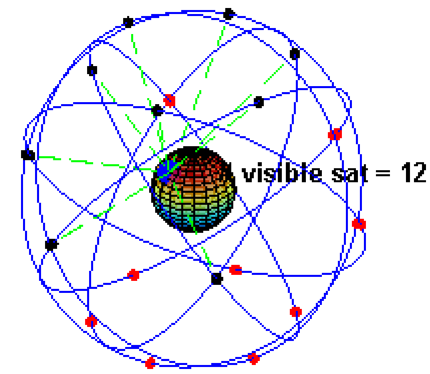
- 3-axes MEMS gyroscope
  - Provides angular velocity
  - Integrate for angular position
- 3-axes MEMS accelerometer
  - Provides accelerations (including gravity)

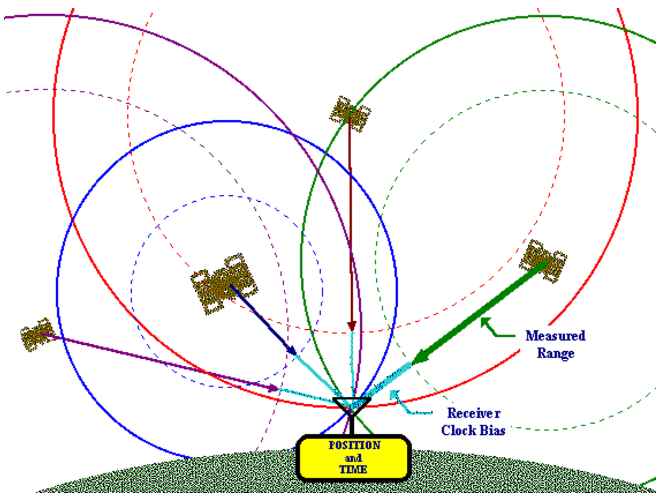


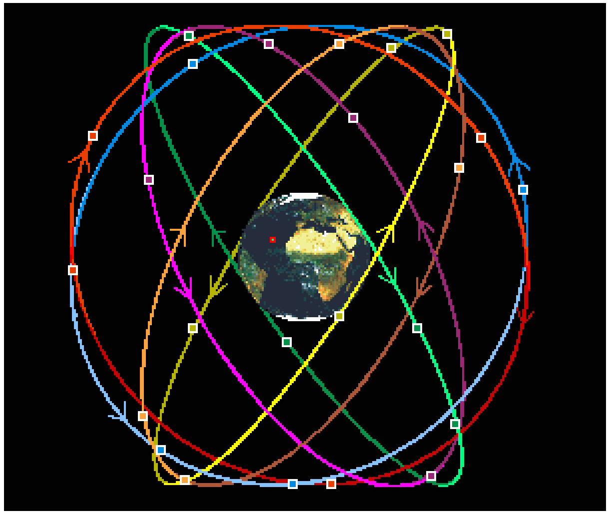


Global Positioning System

- 24+ satellites, 12 hour orbit, 20.190 km height
- 6 orbital planes, 4+ satellites per orbit, 60deg distance
- Every satellite transmits its position and time
- Requires measurements of 4 different satellites
- Low accuracy (3-15m) but absolute





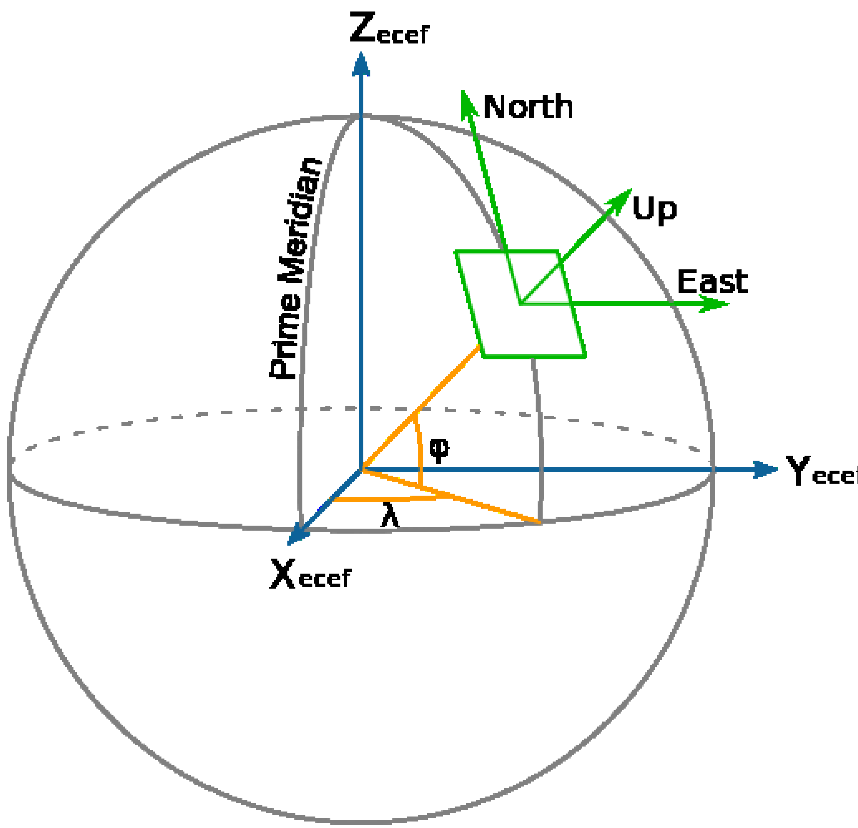


Global Positioning System (Cont.)

- GPS is a space based satellite navigation system that provides location and time information anywhere on earth or near earth where there is direct line of sight with at least four satellites.
- We have to understand some coordinate frame to perform conversion to/from geodetic coordinates (latitude(λ), longitude (Φ)) to local Cartesian coordinates (x, y, z) .

Geodetic to Cartesian Coordinates

- The first frame is a tangent plane (τ) which is tangent to earth surface at some latitude and longitude. X-axis of this plane is pointing north, Y-axis toward east and Z-axis toward the center of earth.
- The second frame is Earth Centered Earth Fixed (ECEF) frame (ϵ) which is used by the GPS and is centered at the center of earth and rotating with earth.



Geodetic to Cartesian Coordinates

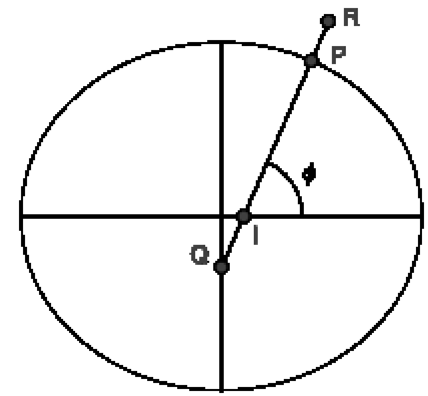
- The X-axis is passing through the prime meridian and the Z-axis is passing through the North Pole, right hand coordinate system.
- The third plane is a Body plane (β) which is attached to the center of mass of the body, it could be NEU (North-East-Up) mostly used by tracker and NED (North-East-Down) mostly used by aero planes.

Geodetic to Cartesian Coordinates

- The conversion from geodetic to Cartesian coordinate is a simple process because there exist a simple and closed form solution.
- The inverse process is complicated because no simple relation relates Φ to X,Y,Z. Broadly there are indirect methods and direct or closed form solution.
- Indirect methods are preferred because of the simplicity.

Geodetic to Cartesian Coordinates

- The earth is considered as a ellipse with semi-major axis $a = 6378137\ m$ and semi-minor axis $b = 6356752.31424\ m$ and $e = 6.694\ 379\ 990 \times 10^{-3}$.
- The coordinates of the GPS receiver in ECEF frame are calculated as follows given geodetic coordi



$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (v(\Phi) + h) \cdot \cos(\Phi) \cdot \cos(\lambda) \\ (v(\Phi) + h) \cdot \cos(\Phi) \cdot \sin(\lambda) \\ \left(v(\Phi) \cdot (1 - e^2) + h\right) \cdot \sin(\Phi) \end{bmatrix}$$

$v(\Phi) = \frac{a^2}{\sqrt{a^2 \cos(\Phi)^2 + b^2 \sin(\Phi)^2}}$ is the radius of curvature in prime vertical plane.

Summary

- Inertial sensors models
  - Euler Angle Representation
  - Accelerometer
    - Inertial Odometry
  - Gyroscope
    - Euler Angle Rate
  - Magnetometer
    - Earth Magnetic Field
  - GPS
    - Geodetic to Cartesian Coordinates

Questions